# AN OPTIMAL CONTROL SOLUTION TO THE CONSTRAINED WEIGHT PORTFOLIO OPTIMISATION PROBLEM WITH CONDITIONING INFORMATION

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Abstract

In the classical discrete-time mean-variance context, a method for portfolio optimisation using conditioning information was introduced in 2001 by Ferson and Siegel [1]. The authors optimise for the unconditional variance of the portfolio given an unconditional level of expected portfolio return and assuming a relationship between lagged signal and risky return.

The fact that there are many possible signals that could be used in this way, and a number of empirical studies that suggest measurable relationships between signal and return, causes this type of portfolio optimisation to be of practical as well as theoretical interest.

Ferson and Siegel obtain analytical formulae for the basic unconstrained portfolio optimisation problem. We show how the same problem, in the presence of a risk-free asset, may be expressed as a general constrained infinite-horizon optimal control problem. Variants of the problem not amenable to closed-form solutions can then be solved using standard numerical optimal control techniques.

Based on a version of the Pontryagin Maximum Principle extended to the doubly-infinite horizon case required to cover our formulation in its greatest generality, we rephrase the previously unsolved constrained-weight variant of the problem in [1] using the optimal control framework and derive the specific necessary conditions applicable. We also show how to recover both the results in [1] and the classical Markowitz solution as special cases of our problem formulation.

Finally, we carry out simulations involving numerical solution of the resulting optimal control problem to assess the extent to which the use of conditioning information brings about practical improvements in the field of portfolio optimisation.

## REFERENCES

[1] Ferson, Wayne E. and Siegel, Andrew F. *The Efficient Use of Conditioning Information in Portfolios.* Journal of Finance, Vol.56 No.3, pp.967-982, 2001.

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